International Space Planes and Hypersonic Systems and Technologies Conferences
6-9 July 2015, Glasgow, Scotland
20th AIAA International Space Planes and Hypersonic Systems and Technologies Conference

10.2514/6.2015-3684



# Mathematical Modeling of RocketMotorTwo

Syed Aseem Ul Islam [*] and Zainab Saleem [†]
*Institute of Space Technology, Islamabad, Pakistan*

**To model the dynamics of a craft like the SpaceShipTwo, which loses almost half its mass in under a minute, knowledge of the variations of the mass, inertia tensor and center of gravity becomes important. Equally important is the knowledge of how the thrust varies during the burn. This paper presents a mathematical model of the RocketMotorTwo which is a hybrid rocket motor powering the SpaceShipTwo. This model has been used not only to obtain an accurate representation of the rocket's thrust profile but also to obtain a profile of how the mass properties of the propulsion system will vary as the rocket motor burns through its HTPB fuel and Nitrous Oxide oxidizer.**



## Nomenclature

| | | |
|---|---|---|
| $\varepsilon$ | = | Nozzle Expansion Ratio |
| $\rho$ | = | Density of HTPB Fuel |
| $\gamma$ | = | Ratio of Specific Heats |
| $\sigma$ | = | Tensile Stress |
| $\lambda$ | = | Nozzle correction factor |
| $A_e$ | = | Nozzle Exit Area |
| $A_p$ | = | Port Cross Sectional Area |
| $A_t$ | = | Nozzle Throat Area |
| $c*$ | = | Characteristic Velocity |
| $d_H$ | = | Hydraulic Diameter |
| $G_o$ | = | Oxidizer Flux |
| $g_0$ | = | Gravity at Sea Level |
| $I_{sp}$ | = | Specific Impulse |
| $M_e$ | = | Exit Mach Number |
| $m_f$ | = | Mass of Fuel |
| $m_o$ | = | Mass of Oxidizer |
| $m_p$ | = | Total Mass of Propellant |
| $N$ | = | Number of Ports in Fuel Grain |
| $p_c$ | = | Chamber Pressure |
| $p_e$ | = | Exit Pressure |
| $P_p$ | = | Port Perimeter |
| $R$ | = | Specific Gas Constant |
| $\dot{r}$ | = | Grain Regression Rate |
| $S_p$ | = | Exposed Surface Area of One Port |
| $T_a$ | = | Adiabatic Flame Temperature |
| $V_e$ | = | Exit Velocity |

## I. Introduction

The SpaceShipTwo is a new, suborbital space plane designed to zoom to an altitude of 110 km above sea level carrying on board two pilots and six passengers (http://www.space.com/17994-how-virgin-galactic-spaceshiptwo-works.html. [Accessed 3 February 2014]). It is powered by a purpose built hybrid rocket motor called

[*] Former Student, Department of Aeronautics & Astronautics, Institute of Space Technology, 1-Islamabad Highway, Islamabad, Pakistan.
[†] Lecturer, Department of Aeronautics & Astronautics, Institute of Space Technology, 1-Islamabad Highway, Islamabad, Pakistan.

the RocketMotorTwo (made by the Sierra Nevada Corporation).There exists little publicly available information for anyone that wants to model either the SpaceShipTwo or the RocketMotorTwo.
Research was carried out that produced data which is reasonably representative of the actual rocket. This data can be used to model and simulate the SpaceShipTwo/RocketMotorTwo.

**Figure 1: SpaceShipTwo with RocketMotorTwo firing (http://www.parabolicarc.com/wp-content/uploads/2013/04/ss2_engine_firing.jpg. [Accessed 23 4 2014]**



## II. Background

Hybrid rocket motors are different from both solid and liquid rockets but combine elements of both[1]. A hybrid rocket consists of a solid fuel grain (much like a solid rocket) but unlike a solid rocket the oxidizer is not binded with the fuel. The oxidizer is pumped through the grain and this mixture is ignited to produce high pressure and temperature gases which are propelled through a nozzle to produce thrust.

Hybrids combine the high specific impulse of solid rockets with the freedom to throttle, shutdown and restart provided by liquid rocket engines. There is difficulty in predicting the regression rate of the grain as it is dependent on chamber pressure, oxidizer flux and even the scale of the motor. Hybrid rocket motors have been known to scale poorly[2]. There are also issues with high frequency instabilities in burning but this and other issues associated with modeling and operating hybrid rockets are beyond the scope of this paper.

This paper focuses on the RocketMotorTwo specifically to produce data which can be used by other researchers. It uses data available publicly and makes intelligent inferences to fill in the missing data.

**Figure 2: RocketMotorTwo undergoing ground testing (http://www.parabolicarc.com/wp-content/uploads/2012/03/RocketMotorTwo_Firing.png. [Accessed 23 4 2014])**

## III. Methodology

It was decided to gather as much data as possible first then make reasonable assumptions to fill in any holes in the vital data that remain. This was then to be used to simulate the firing of the motor with respect to time.
It will be assumed that the composition of the combustion gases will reach an equilibrium value instantaneously (shifting equilibrium model) and using NASA's Chemical Equilibrium with Applications code[3] the concentrations will be calculated at each instant. The code will give values of instantaneous adiabatic flame temperature and instantaneous ratio of specific heats. These values will be used to calculate instantaneous thrust and other parameters.

### A. Known Data

Some information about the rocket motor is provided by the operator of the SpaceShipTwo, Virgin Galactic on their website[‡] while other information was obtained from videos of the firings.

- The fuel and oxidizer are Hydroxyl-Terminated Polybutadiene (HTPB) and Nitrous Oxide [‡, §]
- The rocket can provide up to 270 kN of thrust [§].
- The rocket burns for 56 seconds [**].

### B. Inferred Data

There was a need to obtain further information about the rocket motor. Fortunately much information can be inferred reasonably accurately. The inferences made were:

- The RocketMotorTwo produces 270 kN at vacuum conditions as this is where a rocket produces maximum theoretical thrust.
- The specific impulse is assumed as 280 s for the initial propellant calculations. This value is typical for hybrid motors using the aforementioned fuel and oxidizer[2].
- A 4-port grain configuration was used as this is the configuration used on the SpaceShipTwo's predecessor, the SpaceShipOne[4].
- The density of the HTPB fuel is 960 kg/m$^3$ [2].
- The nozzle always operates as an underexpanded nozzle since during the ascent phase of the SpaceShipTwo the ambient pressure is constantly decreasing.
- Only the momentum thrust is calculated as the pressure thrust will be added in the simulation where the latest ambient pressure conditions are known.

### C. Oxidizer to Fuel Ratio (O/F)

This is an important design parameter which must be selected at the beginning of a hybrid rocket motor design to specify the ratio of fuel and oxidizer that will be needed. The adiabatic flame temperature, molecular weight of combustion products and ratio of specific heats are all functions of O/F ratio. They are also functions of chamber pressure but are very insensitive to it when the value of chamber pressure varies from 25 bar to 75 bar[2].

Using plots for adiabatic flame temperature, molecular mass and ratio of specific heats from Ref.2 the O/F for design was selected as 6.5. This O/F was selected to maximize adiabatic flame temperature and to minimize ratio of specific heats. These optimizations would maximize thrust.

### D. Nozzle

Some simplifying assumptions were made to model the nozzle. It is assumed that the flow is 1 dimensional and isentropic throughout the nozzle. This assumption can be made especially since the nozzle will always be operating in underexpanded conditions.

To calculate the nozzle correction factor, $\lambda$, we referred to a webpage[††] and Ref.5 using them to calculate a $\lambda$ value of 0.985. The throat area was calculated from the following image:

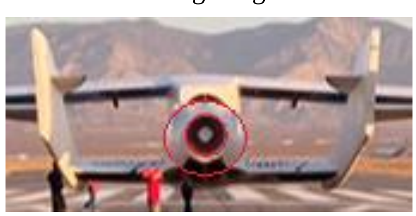

**Figure 3: Diameter of fuselage compared to exit area (http://www.space.com/20869-spaceshiptwo-first-rocket-test-photos.html. [Accessed 23 4 2014])**

Using figure 3 and knowing that the diameter of the fuselage is 2.108 m, the exit diameter was calculated as 1.016 m. This equals an exit area of 0.8107 m$^2$.

[‡] http://www.virgingalactic.com/overview/safety/. [Accessed 3 February 2014]
[§] http://www.spaceflight101.com/spaceshiptwo-first-powered-test-flight.html. [Accessed 3 February 2014]
[**] http://www.youtube.com/watch?v=v1Nvr14smIw. [Accessed 3 February 2014]
[††] http://www.parabolicarc.com/wp-content/uploads/2012/03/RocketMotorTwo_Firing.png. [Accessed 23 4 2014]

From trial an error an expansion ratio of 20 was obtained which produced an ideally expanded nozzle at launch altitude. This means that the throat area is 0.0676 $m^2$.

## E. Grain Diameter

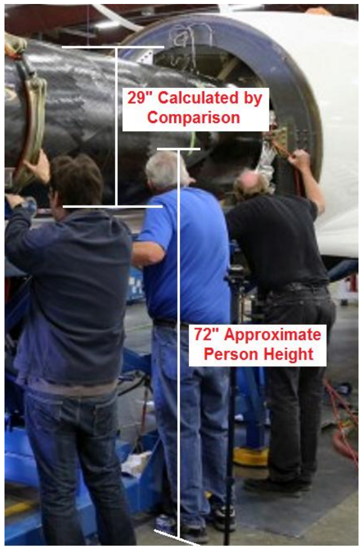


**Figure 4: Estimation of Diameter from image (http://www.wired.com/2013/04/spaceshiptwo-powered-flight/#slideid-57288. [Accessed 23 4 2014])**

To determine the geometry of the grain, the grain diameter had to be estimated. This was done by using another image of the RocketMotorTwo being installed into the SpaceShipTwo and comparing it with the height of an average human being.

The diameter is then estimated to be 0.762 m. This calculates to a radius of 0.381 m. To account for the thickness of the casing the radius of the actual grain was taken to be 0.371 m.

## F. Estimating Delta V Required

Delta V is the theoretical instantaneous change in velocity produced by a rocket motor if all of its thrust was to act instantaneously as an impulse. It is a useful quantity in calculating the mass of propellant required based upon the total mass of the spaceplane.

A simple approach to calculate delta V is used. Using multiple trial runs and analyzing the results of grain length, thrust and burn time a value of 1500 m/s was selected.

## G. Selecting Value of $G_{oi}$

$G_{oi}$ or Initial Oxidizer Flux is a measure of how much (kg/s) oxidizer is flowing per unit area initially. This is an important value to select reasonably. If too much or too little oxidizer flows through the grain cross sectional area the fuel will not burn efficiently. For Nitrous Oxide it is recommended that a value of between 300 $kg/m^2s$ to 800 $kg/m^2s$ be used[1]. A reasonable average value of 500 $kg/m^2s$ was selected for the RocketMotorTwo.

## H. Propellant Flow Rate Estimation

We are now in a position to calculate an estimate for the propellant mass flow rate. This is calculated from the specific impulse as:

$$I_{sp} = \frac{F}{\dot{m}_p g_o} \tag{1}$$

Using $I_{sp}$= 270 s (280 s degraded by 10 to account for real word inefficiencies), $g_o$=9.81 $m/s^2$ and F=270,000 N:

$$\dot{m}_p = \frac{270{,}000}{270 \times 9.81} \tag{2}$$

$$\dot{m}_p = 101.9\ kg/s \tag{3}$$

Now

$$\dot{m}_f = \frac{\dot{m}_p}{1+O/F} \tag{4}$$

$$\dot{m}_f = \frac{101.9}{1+6.5} \tag{5}$$

$$\dot{m}_f = 13.6\ kg/s \tag{6}$$

$$\dot{m}_p = \dot{m}_f + \dot{m}_o \tag{7}$$

$$101.9 = 13.6 + \dot{m}_o \tag{8}$$

$$\dot{m}_o = 88.3\ kg/s \tag{9}$$

## I. Grain Geometry

At this point enough information is known to specify a grain geometry that will be used for future simulations. Using the value of $G_{oi}$, oxidizer mass flow rate and maximum grain radius as constraints together with the knowledge of the port configuration[4] the following port geometry was obtained:

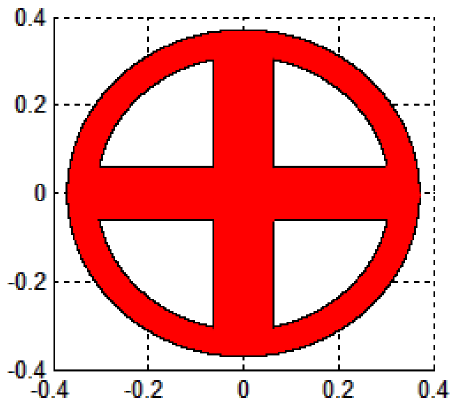


**Figure 5: Grain Geometry (in meters)**

Other grain geometries tried included tubular, star and geometries with other number of ports but these had trouble satisfying the constraints of $G_{oi}$, oxidizer mass flow rate and maximum grain radius while being able to produce the required maximum thrust level with the inferred burn time.

## J. Grain Regression Rate

A model used by Ref.2 which has been shown to agree well with experimental results for HTPB/N20 motors is used.

$$\dot{r} = aG_o{}^n d_H{}^m \tag{10}$$

Where a and m are regression coefficients and $d_H$ is the hydraulic diameter of each port. With the suggested values the regression rate expression becomes:

$$\dot{r} = 0.0000637 \times {G_o}^{0.41} {d_H}^{-0.24} \tag{11}$$

### K. RocketMotorTwo Code Algorithm

At this point a rocket simulator code was written in MATLAB which works on the following algorithm using a constant oxidizer to fuel ratio:

1) Initialize temperature, pressure, ratio of specific heats, molecular mass, specific gas constant, O/F and exit Mach number.
2) Calculate current hydraulic diameter of port.

$$d_H = \frac{4A_P}{P_P} \tag{12}$$

3) Calculate current regression rate of fuel from Eq. (11).
4) Calculate exposed grain area.
5) Calculate fuel mass flow rate from exposed area and grain regression rate information.

$$\dot{m}_f = N\dot{r}\rho S_p \tag{13}$$

6) Calculate oxidizer mass flow rate by assuming a constant O/F of 6.5.
7) Calculate total propellant mass flow rate from fuel mass flow rate and oxidizer mass flow rate from Eq. (7).
8) Using NASA CEA Code calculate adiabatic flame temperature, ratio of specific heats and molecular mass at current value chamber pressure and O/F ratio
9) Calculate characteristic velocity

$$c^* = \frac{\sqrt{\gamma R T_a}}{\gamma \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}} \tag{14}$$

10) Calculate chamber pressure from characteristic velocity and propellant mass flow rate and throat area.

$$P_c = \frac{\dot{m}_p c^*}{A_t} \tag{15}$$

11) Calculate Exit Mach number from ratio of specific heats and expansion ratio.

$$M_e = \frac{1}{\varepsilon} \sqrt{\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} {M_e}^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}} \tag{16}$$

12) Calculate exit pressure from exit Mach and ratio of specific heats using isentropic flow equations.

$$p_e = \frac{p_c}{\left(1+\frac{\gamma-1}{2}{M_e}^2\right)^{\frac{\gamma}{\gamma-1}}} \tag{17}$$

13) Calculate exit velocity.

$$V_e = \sqrt{\frac{2\gamma R T_a}{\gamma - 1}\left(1 - \frac{p_e}{p_c}^{\frac{\gamma-1}{\gamma}}\right)} \tag{18}$$

14) Calculate thrust.

$$F = \lambda \dot{m}_p V_e \tag{19}$$

15) Calculate $I_{sp}$.

$$I_{sp} = \frac{F}{\dot{m}_p g_0} \tag{20}$$

16) Reduce web thickness according to time step and regression rate.
17) Add to cumulative sum for mass of fuel and oxidizer consumed.
18) Advance time.
19) Go to step 2.
20) Exit loop when fuel has burnt through the entire web thickness.

After simulation the following data was obtained:

**Table 1**

**Burn Simulation Results**

| | |
|---|---|
| $m_f$ | 562.1 kg |
| $m_o$ | 3,653.6 kg |
| $m_p$ | 4,215.7 kg |
| Burn time | 54.85 s |
| $m_{final}$ (SS2) | 5,524.0 kg |
| $m_{initial}$ (SS2) | 9,740.0 kg |
| $L_p$ | 2.172 m |

## L. Calculation of Mass, Center of Gravity and Inertia Tensor variation

After having run the simulation for the duration of the burn the following data was extracted:

- Total mass of oxidizer needed to propagate burn at a constant O/F.
- Variation of the web thickness of the HTPB grain.
- Variation of the mass of oxidizer expended.

- Maximum chamber pressure reached.

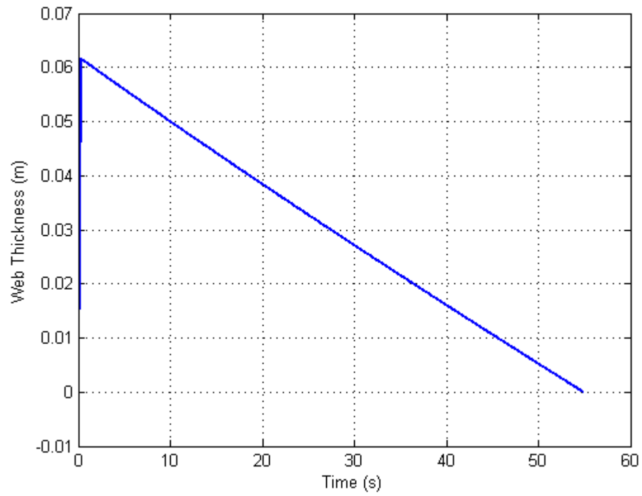


**Figure 6: Variation of Grain Web Thickness with Time**

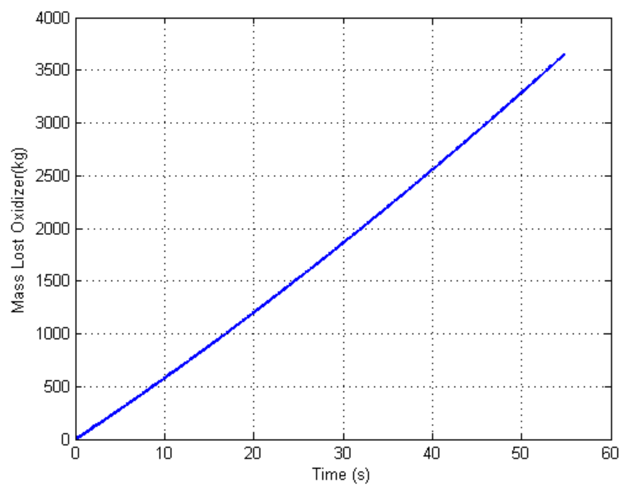


**Figure 7: Variation of Oxidizer Mass Lost**

Next the geometry of the oxidizer tank and the CTN (Casing Throat Nozzle) must be defined.

1. *Modelling the Oxidizer Tank*

From the Figure 8 it can be observed that the oxidizer tank is of a cylindrical shape with curved capped ends.

**Figure 8: Oxidizer Tank (http://www.eejournal.com/files/cache/74cb8b0fed6f63cd4a318b592df8284f.jpeg. [Accessed 23 4 2014])**



It was assumed that the tank is simple cylinder and that the maximum tensile stress that will occur due to the pressurization will be the hoop stress. Therefore any thickness of material decided based upon the hoop stress will ensure safe operation.

To calculate the mass of the oxidizer tank the thickness of the shell, the pressure and the radius must be known. The radius of the tank is assumed to be 1.06 m, 8 cm shy of the radius of the fuselage which is 1.14 m.

According to Ref. 6 the oxidizer tank of the SpaceShipOne was pressurized to 50 bar. It is assumed that SpaceShipTwo's tank is pressurized to a similar pressure.

The density of Nitrous Oxide (907.02 kg/m$^3$ at 48 bar[7]) and the mass of oxidizer from the simulation (3,653.6 kg) was used to calculate a tank volume of 4.03 m$^3$. This allowed the calculation of the tank length of 1.27 m using the known radius.

Using the theory of thin shells, hoop stress for a thin pressure vessel of circular cross section can be expressed as:

$$\sigma = F.S \times \frac{Pr}{t} \tag{21}$$

Where P is pressure, r is the radius of vessel and t is the shell thickness.

To obtain the maximum tensile stress a typical high strength carbon fibre/epoxy laminate was selected with a tensile strength of 1,240 MPa and a density of 1550 kg/m$^3$ [8]. It is known that composites are used to construct this tank. Then from Eq. (21) we can obtain:

$$t = F.S \times \frac{Pr}{\sigma}$$

Taking the factor of safety as 2:

$$t = 2 \times \frac{4.8 \times 1.06}{1240}$$

$$\boldsymbol{t = 8.2\ mm}$$

Using the obtained data the tank mass was calculated as 174 kg and a computer model was constructed.

2. *Modelling of the CTN*

The radius of fuel grain has been previously estimated as 0.371 m. From the simulation the maximum chamber pressure reached is 35 bar. Again using the same thin shell theory, same material and a factor of safety of 2 we have:

$$t = 2 \times \frac{3.5 \times 0.371}{1240}$$

$$\boldsymbol{t = 2.1\ mm}$$

It is known that multiple layers of composites are used in the construction of the RocketMotorTwo and therefore the thickness used is three times the one calculated.

For the length of the CTN the injector and mixing chamber volume had to be taken into account. According to Ref. 9 the length to diameter ratio for the injector can be assumed to be 0.5 while for the mixing chamber it can be assumed to be 1. From this the lengths of the injector assembly and mixing chamber were calculated to be 0.371 m 0.742 m respectively. A computer model of the CTN was also constructed.

### 3. *Dealing with Fuel and Oxidizer Masses*

To account for the variation of the mass and mass distribution of the fuel and oxidizer computer models of the fuel grain and the internal volume of the oxidizer tank were generated. Appropriate densities were assigned for when the rocket is initially loaded.

Using the web thickness data (Figure 6) the fuel grain computer model was regenerated at various time steps. Similarly using the oxidizer loss data (Figure 7) the density of the nitrous oxide in the tank was recalculated at various time steps. Here it is assumed that all of the Nitrous Oxide present inside at any point in time is a single phase, homogenous fluid. Strictly speaking this assumption is invalid but it can be assumed for the calculation of mass properties.

### 4. *Mass and Mass Properties*

Combining the various generated computer models the mass, center of gravity and the inertia tensor were calculated at selected points of time. These models depict the state of the propulsion system at various points in time along the burn history.

As analytic formulae to calculate mass, center of gravity and inertia tensor for a complicated geometry such as that of the propulsion system can be rather involved a simpler approach of fitting curves to the various data points was used. This produced functions of time which give the mass, the location of the center of gravity and the three principal moment of inertias. The center of gravity is calculated using standard aircraft right handed coordinate system centered at the front face of the oxidizer tank.

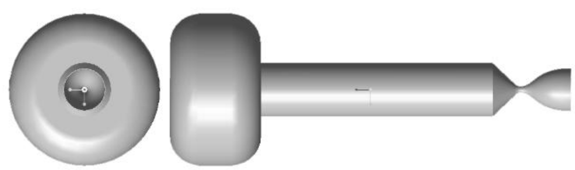

**Figure 9: Orthogonal View of Combined Assembly**

As the geometry is axisymmetric the principal momenta of inertia are calculated about a coordinate system centered at the center of gravity aligned with the aircraft axes, all off diagonal moment of inertia are zero and $I_y=I_z$. The curve fit equations for t<=54.85 s are presented below. For t>54.85 s the values of the equations at t=54.85 s must be taken as constants as this is where the burn has ended.

$$m\ [kg] = 0.0001738t^4 - 0.02507t^3 + 1.009t^2 + 87.05t + 4679 \quad (22)$$

$$x_{cg}\ [\mathrm{m}] = -0.9536\mathrm{e}^{0.001542\mathrm{t}} - 0.00001411e^{0.1899t} \quad (23)$$

$$I_x\ [kg.m^2] = 0.0001025t^4 - 0.01442t^3 + 0.5854t^2 - 42.03t + 2161 \quad (24)$$

$$I_y\ [kg.m^2] = I_z = -0.000103t^4 + 0.005334t^3 - 0.1234t^2 - 63.2t + 4671 \quad (25)$$

### M. Results

The following data was obtained by plotting all the data genereated from previous sections against time.

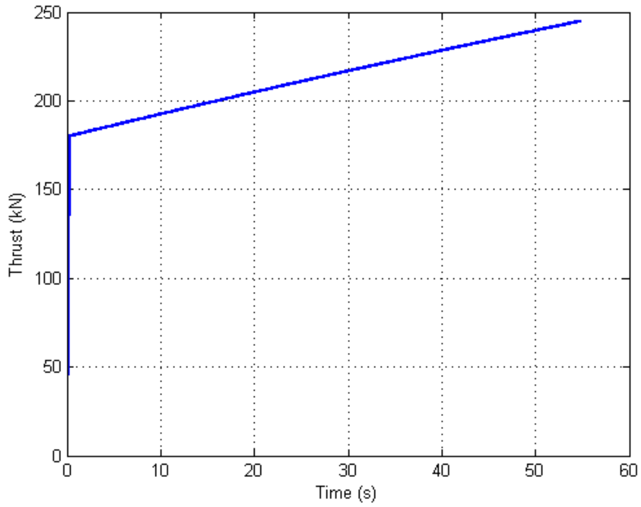


**Figure 13: Momentum Thrust**

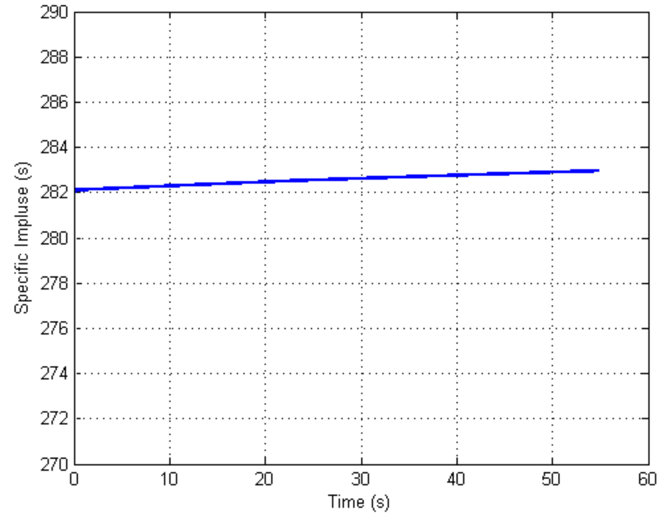


**Figure 10: Specific Impulse**

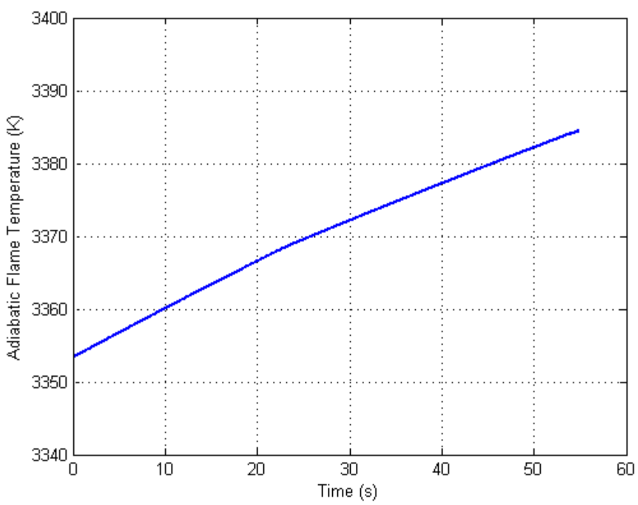


**Figure 12: Adiabatic Flame Temperature**

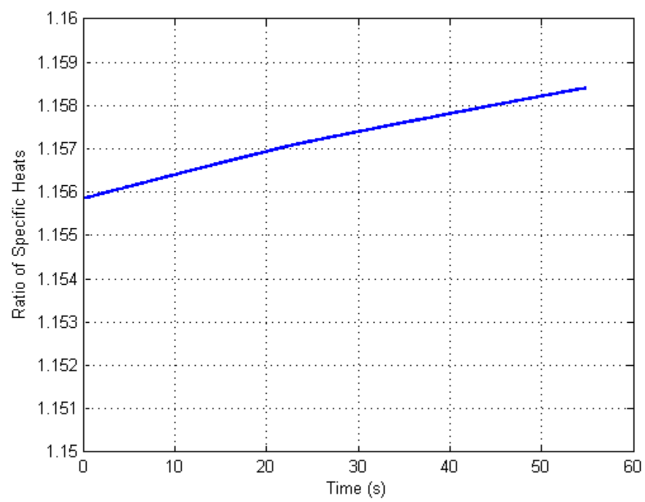


**Figure 11: Ratio of Specific Heats**

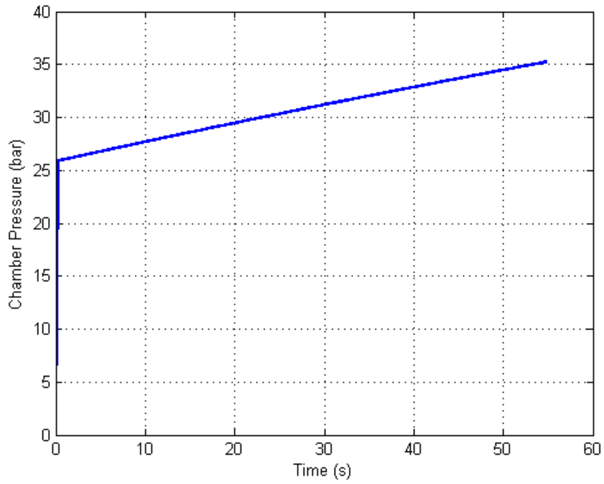


**Figure 17: Chamber Pressure**

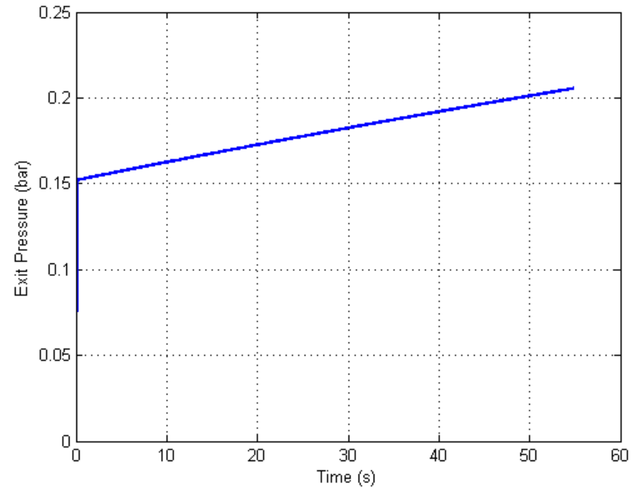


**Figure 16: Exit Pressure**

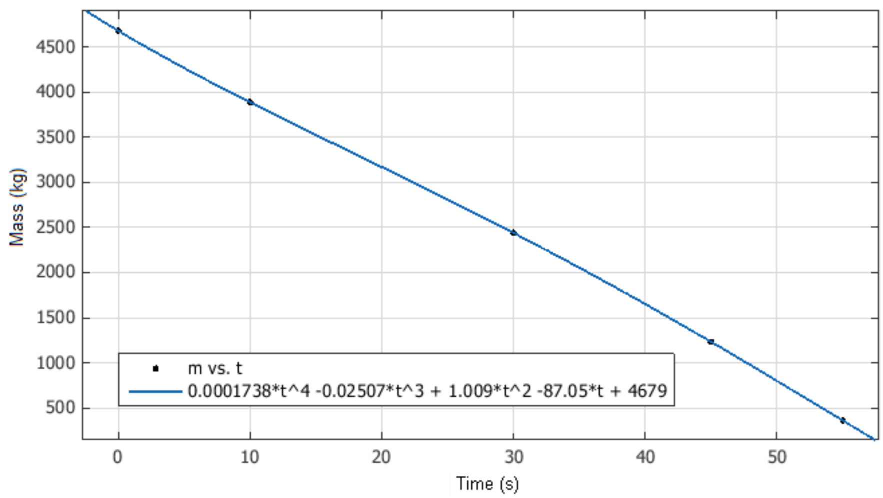


**Figure 15: Curve Fit for Mass**

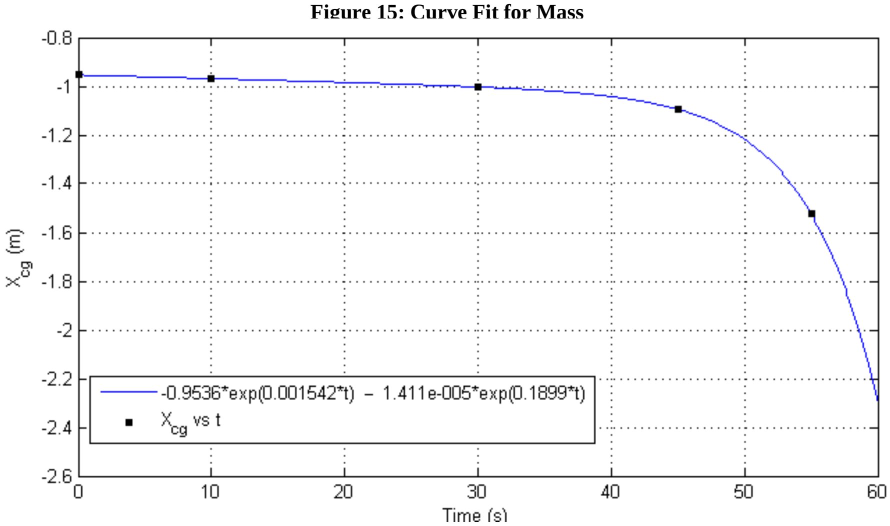


**Figure 14: Curve Fit for $X_{cg}$**

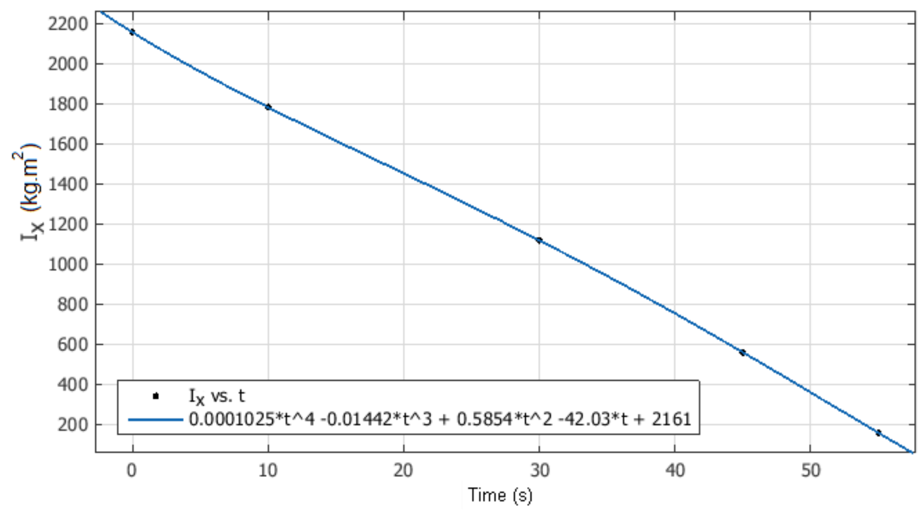


**Figure 19: Curve Fit for $I_x$**

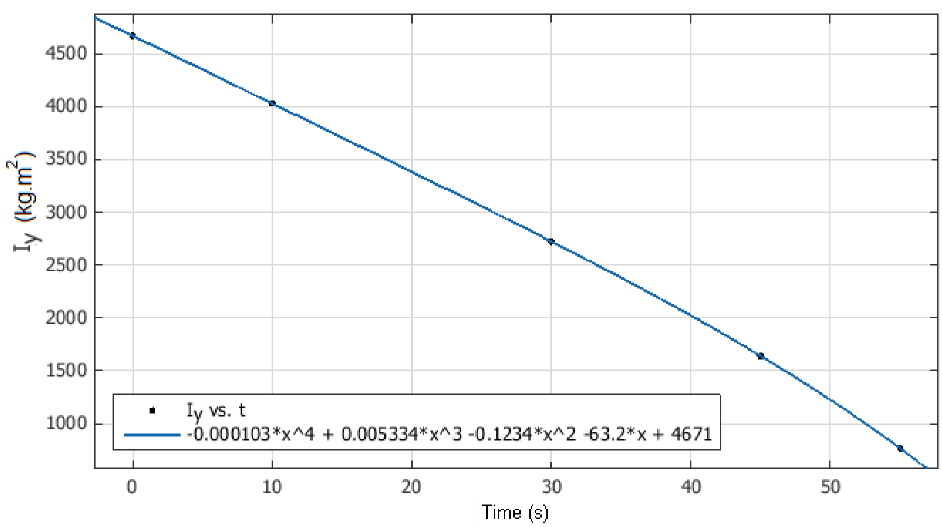


**Figure 18: Curve Fit for $I_y$ and $I_z$**

## IV. Conclusion

This research paper produced data that can be seen to agree well with what is known about the SpaceShipTwo. The maximum thrust is around 270 kN(when the pressure thrust is added to the momentum thrust) agreeing well with known value[‡‡], the burn time is very close to 56 seconds (also agreeing well with known value[§§]) and the length of the motor ($L_p$) came out as close to that of the RocketMotorTwo as it appears in photographs (around 2 meters). This leads us to the conclusion that the thrust profile presented in this paper is a reasonably accurate representation of the actual RocketMotorTwo. The value of thrust can easily be corrected for the presence of ambient pressure for any value of burn time so it can be used in simulations for the SpaceShipTwo. Additionally this confidence in the thrust data allows us to calculate reasonably representative mass data.

‡‡ http://www.spaceflight101.com/spaceshiptwo-first-powered-test-flight.html. [Accessed 3 February 2014]
§§ http://www.youtube.com/watch?v=v1Nvr14smIw. [Accessed 3 February 2014]

## Acknowledgments

The authors of this paper are grateful to the Institute of Space Technology for their resources and to Dr.Ali Kamran for his guidance during the course of this research.

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